

B.Sc. EXAMINATION BY COURSE UNITS

MAS125 Calculus II — Sample Exam Paper

This is a sample paper showing the format of the exam.

The duration of this examination is 2 hours.

You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Candidates must not remove the question paper from the examination room.

**YOU ARE NOT PERMITTED TO START READING THIS QUESTION
PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR**

1. Marks are awarded for partial answers, so you should show your workings.

- (a) [7 marks] Find the locus of points $z = x + iy$ in the Argand diagram that satisfy the condition $|z - 2| = 3$ and interpret this geometrically.
- (b) [7 marks] Find the 5th roots of the complex number $16\sqrt{2}(1 - i)$ and plot these in an Argand diagram.
- (c) [7 marks] Find the sum of the series:

$$\sum_{n=1}^{\infty} \frac{10}{(5n-3)(5n+2)}.$$

- (d) [7 marks] Find the first four terms of the binomial series for $(1 + x^3)^{-1/4}$.
- (e) [7 marks] Find the radius and interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{x^n}{n4^n}$$

being careful to specify the behaviour at the endpoints of the interval.

- (f) [7 marks] Find all second-order partial derivatives of the function $f(x, y) = xe^y + y + 1$.
- (g) [7 marks] Find the equations for the tangent plane and normal line at the point $P_o(1, -1, 3)$ for the surface $x^2 + 2xy - y^2 + z^2 = 7$.
- (h) [7 marks] Find the Jacobian $\partial(x, y)/\partial(u, v)$ for the transformation $x = u \sin v, y = u \cos v$.

[Next question overleaf]

2. [11 marks] Use L'Hopital's rule to prove that as $n \rightarrow \infty$

$$\left(1 + \frac{x}{n}\right)^n \rightarrow e^x.$$

Deduce the limit of

$$\left(\frac{2n+1}{3n-1}\right)^n.$$

3. [11 marks] Use the Lagrange multiplier method to find the extreme values of the function $f(x, y, z) = x - y + x$ on the unit sphere $x^2 + y^2 + z^2 = 1$.
4. [11 marks] Find the linearization of the function $f(x, y, z) = x^2 - 3xy + 5$ at $P_0(2, 1)$. Hence find an upper bound for the error E in approximating $f(x, y, z)$ by $L(x, y, z)$ over the rectangle $|x - 2| \leq 0.1$, $|y - 1| \leq 0.1$.
5. [11 marks] Sketch the region of integration for the double-integral

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx.$$

By reversing the order of integration, evaluate this integral.